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# Single-spin asymmetry in deeply virtual Compton scattering: Fragmentation region of polarized lepton

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For the kinematical region when a hard photon is emitted predominantly close to the direction of motion of a longitudinally polarized initial electron and relatively small momentum is transferred to a proton we calculate the azimuthal asymmetry of photon emission. It arises from the interference of the Bethe-Heitler amplitude and amplitudes described by a heavy photon impact factor. Azimuthal asymmetry does not decrease in the limit of infinite c.m.s. energy. The lowest order expression for the impact factor of a heavy photon is presented.

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## I. INTRODUCTION

At present the process of deeply virtual Compton scattering (DVCS) in lepton-nucleon collisions at high energies is of firsthand interest to theorists [1–8] as well as to experimenters [9] investigating the long-standing problem of the nucleon spin content. Among others it was realized that leptoproduction of real photons off nucleons could shed light on the so-called spin crisis problem by allowing for a direct measurement of off-forward parton distributions. Indeed a decomposition of the nonforward Compton scattering amplitude for the case in which one of the photons is on mass shell and another one is off contains 15 structure functions with at least 4 of which could be put to the test [1–8]. Their first moments determine the Dirac, Pauli, axial-vector, and pseudoscalar form factors of a proton while their second moments are related (in the forward limit) to a proton spin fraction carried by gluons and quarks and the orbital momenta of the latter. These structure functions could be tested in deep inelastic scattering (DIS) experiments with the longitudinally polarized initial lepton aimed at measuring the azimuthal correlation between a real photon and scattered lepton. There are two mechanisms of photon emission: namely, emission off leptons (Bethe-Heitler) and quark lines. The last option may be split up into two gauge-invariant sets: the emission off a proton valence quark and emission from a quark-antiquark pair produced by a virtual photon and a gluon (Fig. 1). For the case of small angle scattering (the limit of small Bjorken variable $x_{Bj}$) and the limit of high c.m. system (c.m.s.) energies the leading nonvanishing contribution arises from the impact factor (IF) mechanism, the amplitude of virtual photons conversion into real photons in the gluonic field of a proton (for more details see [4,8]). It should be stressed that we deal with a kinematical region of lepton fragmentation which means that the invariant mass of the scattered lepton and real photon is much less than the invariant variable $s$: $(p_2+k_1)^2 \ll s$. In this region both Bethe-Heitler (BH) and IF amplitudes do not fall with $s$ increasing which is ensured by a $t$-channel photon (BH case) or two-gluon (IF case) exchange, whereas the contribution corresponding to the so-called *handbag* diagram [5] falls with $s$ and dominates in the region $(p_2+k_1)^2 \lesssim s$. The effect of azimuthal correlation appears as the interference of a real BH amplitude with a purely imaginary one of an IF mechanism of real photon creation. The interference is not zero due to the purely imaginary spin density matrix of a polarized lepton.

The azimuthal asymmetry has the simplest form $\mathcal{A} = \Delta|M|^2/|M_{BH}|^2 \sim \sin\phi$, where $\phi$ is the azimuthal angle between the planes formed by the momenta of initial and scattered leptons and of initial leptons and photons. It was shown that the higher harmonics in the Fourier decomposition of the asymmetry are related to the structure functions mentioned above. The contribution derived here is sensitive only to the gluon density $zg(z,Q^2)$ inside a proton. For small values of the energy fraction $z$ carried by gluons, one has $zg(z,Q) \approx 6Q^2$ [GeV$^2$], $Q^2 \sim 1$ GeV$^2$ [10].

In what follows we study the case when the initial proton is unpolarized and the final states are a scattered lepton, a recoil proton, and a hard photon from the fragmentation region of initial leptons. Furthermore, the lowest order contribution ($\sim \alpha_s^2$) to the asymmetry is investigated. The higher

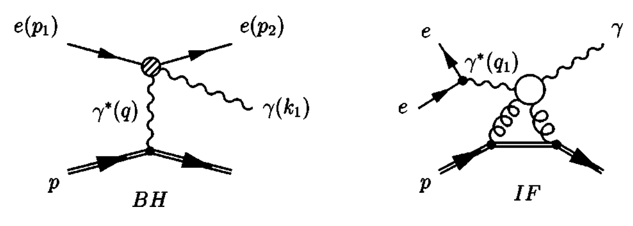


FIG. 1. Feynman diagrams corresponding to BH and IF amplitudes (the crossed diagram for the IF is implied).

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PT effects were considered in Ref. [8] and took into account the Balitskii-Fadin-Kuraev-Lipatov (BFKL) ladder.

## II. BETHE-HEITLER AMPLITUDE

Let us consider the radiative electron-proton scattering

$$e(p_1,\xi)+P(p)\rightarrow e(p_2)+P(p')+\gamma(k_1),$$

where we indicate in parentheses the four-momenta of particles, and $\xi$ is the degree of the longitudinal polarization of electron. We will restrict ourselves to the kinematics when the absolute magnitude of a square of the momentum transfer between initial and final state electrons is small with respect to the c.m.s. energy squared:

$$s=(p_1+p)^2\gg Q_1^2=-(p_1-p_2)^2,$$
$$-t=Q^2=-q^2\gg p_1^2=p_2^2=m_e^2,$$
$$p^2=p'^2=M^2,\quad q=p'-p. \tag{1}$$

Note that we use notation typical for calculations based on Sudakov parametrization of four-vectors. The invariants do not always coincide with those routinely used in data analysis. Thus our $x$, $Q_1^2=\boldsymbol{p}_2^2/x$, and $Q^2$ correspond to $1-y$, $Q^2$, and $-t$, adopted by experimental collaborations; $y$ is a scaling variable.

The leading nonvanishing in the limit of large $s$ contribution arises from the two Feynman amplitudes. One of them, the so-called Bethe-Heitler amplitude, describing hard photon emission by an electron blob, has the following form:

$$M_\lambda^{BH}=\frac{(4\pi\alpha)^{3/2}}{q^2}\overline{u}(p_2)O_{\mu\sigma}u(p_1,\xi)\overline{u^\lambda}(p')\times V_\nu u^\lambda(p)g^{\mu\nu}e^\sigma(k_1)$$
$$=\frac{(4\pi\alpha)^{3/2}}{q^2}\left(\frac{-2x_1}{sdd_1}\right)\overline{u}(p_2)v_\sigma u(p_1,\xi)\times e^\sigma(k_1)sN^\lambda, \tag{2}$$

with

$$V_\nu=\gamma_\nu F_1(Q^2)+\frac{[\gamma_\nu,\hat{q}]}{2M}F_2(Q^2),$$

$$N^\lambda=\frac{1}{s}\overline{u^\lambda}(p')\left(\hat{p}_1F_1(Q^2)+\frac{\hat{q}\hat{p}_1}{M}F_2(Q^2)\right)u^\lambda(p),$$

$$\sum_\lambda |N^\lambda|^2=2F(Q^2),$$

$$F(Q^2)=F_1^2(Q^2)+\frac{Q^2}{M^2}F_2^2(Q^2). \tag{3}$$

Here $\lambda=\pm1$ describes a proton chiral state, $F_{1,2}$ are the Dirac and Pauli form factors, $M$ is the proton mass, $e(k_1)$ is the photon polarization vector, and

$$v_\sigma=sx(d-d_1)\gamma_\sigma+xd_1\gamma_\sigma\hat{q}\hat{p}+d\hat{p}\hat{q}\gamma_\sigma,$$

the effective vertex describing the Compton scattering [11]. The quantities

$$d=xx_1[(p_1-q)^2-m_e^2],\quad d_1=-x_1[(p_1-k_1)^2-m_e^2],\quad q^2$$

can be reexpressed using the Sudakov decomposition of the four-vectors:

$$d=x_1^2m_e^2+(\boldsymbol{k}_1+x_1\boldsymbol{q})^2,\quad d_1=x_1^2m_e^2+\boldsymbol{k}_1^2,\quad Q^2=-q^2=\boldsymbol{q}^2,$$

where $\boldsymbol{k}_1,\boldsymbol{p}_2,\boldsymbol{q}$ are the two-dimensional components of the photon, scattered electron, and recoil proton momenta in the plane transverse to the beam axis. They obey the conservation law $\boldsymbol{k}_1+\boldsymbol{p}_2+\boldsymbol{q}=0$. Here $x,x_1$ are the energy fractions of the scattered electron and real photon satisfying $x+x_1=1$. The squared matrix element summed over polarization states and the cross section can be brought to the form [11]

$$\sum |M^{BH}|^2=2^{11}\pi^3\alpha^3\frac{s^2}{\boldsymbol{q}^2}\frac{x_1^2x(1+x^2)}{dd_1}F(Q^2),$$

$$\mathrm{d}\sigma_{BH}^{eP\rightarrow(e\gamma)P}=\frac{2\alpha^3x_1(1+x^2)}{\pi^2\boldsymbol{q}^2dd_1}F(Q^2)\mathrm{d}^2\boldsymbol{k}_1\mathrm{d}^2\boldsymbol{q}\mathrm{d}x. \tag{4}$$

It is important to note that the amplitude $M^{BH}$ is real.

## III. ASYMMETRY EVALUATION

Consider now the two-loop level correction to the amplitude studied above, describing emission of a hard photon from the intermediate state of a pair of charged quarks created by the virtual photon and converted to the real one through the two gluons exchange. The corresponding amplitude differs from the QED one only by the factor $C=\Sigma Q_q^2$ ($Q_q$ is a quark charge in units of $e$) and the gluon density factor $G(z,k,Q)=z\mathrm{d}g(z,k,Q)/\mathrm{d}\ln Q^2,k^2\sim Q^2\ll s$ (see Ref. [10]). The amplitude of the IF mechanism is purely imaginary and may be expressed in terms of the photon IF:

$$M^{IF}=4C\alpha_s^2\alpha\frac{\mathrm{i}s(4\pi\alpha)^{1/2}}{q_1^2}\overline{u}(p_2)\gamma_\mu u(p_1,\xi)N^\lambda\times\int\frac{\mathrm{d}^2\boldsymbol{k}G(z,k,Q)}{\pi\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\frac{\mathrm{d}^2\boldsymbol{q}_+\mathrm{d}x_+}{\pi x_+x_-}I_{\mu\sigma}e^\sigma(k_1),$$

$$q_1^2=-\frac{\boldsymbol{p}_2^2}{x}, \tag{5}$$

where the tensor $I_{\mu\sigma}$ is given through the tensor of elastic gluon-photon scattering,

$$I_{\mu\sigma}=\frac{p^{\alpha}p^{\beta}}{s^2}T_{\alpha\beta\mu\sigma}. \tag{6}$$

Its explicit form is presented in Appendixes A and C. In the last appendix we derive the heavy photon IF with both photons off mass shell.

The relevant expression for the contribution to the cross section reads

$$\Delta|M|^2=\sum 2M^{IF}(M^{BH})^*$$

$$=s^2\xi 2^{11}C\frac{xx_1\pi^2}{\boldsymbol{q}^2\boldsymbol{p}_2^2 dd_1}\alpha^3\alpha_s^2$$

$$\times\int\frac{d^2\boldsymbol{k}G(k,Q,z)}{\pi\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\int\frac{d^2\boldsymbol{q}_+ dx_+}{x_+x_-\pi}J\cdot F_1(Q^2),$$

$$J=\frac{i}{s}I_{\mu\nu}L_{\mu\nu},\qquad L_{\mu\nu}=\frac{1}{4}\mathrm{Tr}[\hat{p}_2 v_\nu \hat{p}_1\gamma_\mu\gamma_5]. \tag{7}$$

Using the gauge invariance conditions $T_{\alpha\beta\ldots}k_\alpha=T_{\alpha\beta\ldots}(q-k)_\beta=0$ we can make the following replacement in the expression for $I_{\mu\sigma}$:

$$\frac{p_\alpha p_\beta}{s^2}\rightarrow\frac{k_\alpha^{\perp}(q-k)_\beta^{\perp}}{\tilde{s}s_1'},$$

$$\tilde{s}=\frac{1}{x_1}[(q_++q_-)^2+(\boldsymbol{q}_1+\boldsymbol{k})^2],$$

$$s_1'=\frac{1}{x_1}[(q_++q_-)^2+(\boldsymbol{k}_1+\boldsymbol{k}-\boldsymbol{q})^2]. \tag{8}$$

The next step is to perform the $d^2\boldsymbol{k}$ integration. We suppose that small values of $|\boldsymbol{k}|$ dominate as this region is enhanced by the factor $zg(z,|\boldsymbol{k}|)$. Then the integration could be carried out as follows:

$$\int\frac{d^2\boldsymbol{k}}{\pi\boldsymbol{k}^2\boldsymbol{k}'^2}\boldsymbol{k}^i\boldsymbol{k}'^jG(z,\boldsymbol{k},\boldsymbol{k}')$$

$$=\int_0^1 dx\int\frac{d^2\boldsymbol{k}\boldsymbol{k}^i(\boldsymbol{q}-\boldsymbol{k})^jG(z,\boldsymbol{k},\boldsymbol{q}-\boldsymbol{k})}{\pi[(\boldsymbol{k}-x\boldsymbol{q})^2+Q^2x(1-x)]^2}$$

$$\approx\frac{1}{2}\delta^{ij}\int_0^1 dx\,zg(z,xQ,(1-x)Q)\approx\frac{1}{2}\delta^{ij}zg(z,Q/2). \tag{9}$$

Thus to the accuracy of approximately 10% (with $Q^2$ of a few $\mathrm{GeV}^2$) we may put $|\boldsymbol{k}|=|\boldsymbol{k}'|=Q/2$ in the nonsingular part of the integrand.

It should be noted that only the structure

$$E=(p,p_1,p_2,q)=\varepsilon_{\alpha\beta\gamma\delta}p^\alpha p_1^\beta p_2^\gamma q^\delta=\frac{s}{2}[\boldsymbol{p}_2\times\boldsymbol{q}]_z \tag{10}$$

survives integrations over $d^2\boldsymbol{q}_+$, $dx_+$ (for details see Appendixes A,B).

The single-spin asymmetry is defined as

$$\mathcal{A}=\frac{d\sigma^{\uparrow}-d\sigma^{\downarrow}}{d\sigma^{\uparrow}+d\sigma^{\downarrow}}, \tag{11}$$

where the arrows refer to the electron polarization, and $d\sigma$ stands for the total differential cross section, $d\sigma=d\sigma_{BH}+d\sigma_{INT}$. This asymmetry is related ($\mathcal{A}=\mathcal{A}_\phi\sin\phi$) to the azimuthal one given by

$$\mathcal{A}_\phi=\frac{\int_0^{\pi}d\phi\,d\sigma-\int_{\pi}^{2\pi}d\phi\,d\sigma}{\int_0^{2\pi}d\phi\,d\sigma}, \tag{12}$$

which is sometimes simpler for measurements.

It is easy to see that the definition of the asymmetry given in Eq. (11) coincides with the one given in the Introduction and is found to be

$$\mathcal{A}=C\xi\frac{\alpha_s^2}{\pi}\frac{zg(z,Q/2)|_{z\to 0}F_1(Q^2)}{F(Q^2)}\left|\frac{\boldsymbol{q}}{\boldsymbol{p}_2}\right|\Phi(x)\sin\phi,$$

$$\Phi(x)=-\frac{1+x}{3[1+x^2]}\left(2\ln\frac{\boldsymbol{p}_2^2}{m^2}-1\right). \tag{13}$$

Here $Q=|\boldsymbol{q}|$ is the momentum transfer to a proton which bears to a some extent a latent dependence on an energy fraction of the scattered lepton, $|\boldsymbol{p}_2|$ is the transverse component of the scattered electron momentum, and $m=0.3$ GeV is the quark constituent mass. In addition it has been assumed that $\boldsymbol{p}_2^2\gg Q^2\gg m^2$ and terms of order $(m^2/\boldsymbol{p}_2^2)\ln(\boldsymbol{p}_2^2/m^2)$ have been dropped for their subleading nature (which gives an accuracy of the derivation of $\sim$10%).

Equation (10) could be slightly rewritten to take the form

$$E=-(p,p_1,q_1,q)=-\frac{s}{2}[\boldsymbol{q}_1\times\boldsymbol{q}]_z,$$

which gives us an azimuthal angle between lepton and virtual-real photon planes. The corresponding asymmetry is often used in theoretical and experimental papers.

## IV. DISCUSSION AND CONCLUSION

As was shown in Ref. [12] at high virtualities of the virtual $\gamma^*$ one can apply the perturbative QCD (pQCD) factorization in order to study the process of hard exclusive electroproduction of vector mesons. The nonperturbative information related to the proton is described by the matrix element of a two-gluon operator approximated by the gluon distribution function. In DVCS, as was emphasized by Ji [2], one can measure the asymmetric gluon distribution. In our case one has a small momentum transfer to the nucleon and the approach of Ref. [12] is valid. Moreover, the quark-gluon mixing in the evolution can be ignored. In diffractive kine-

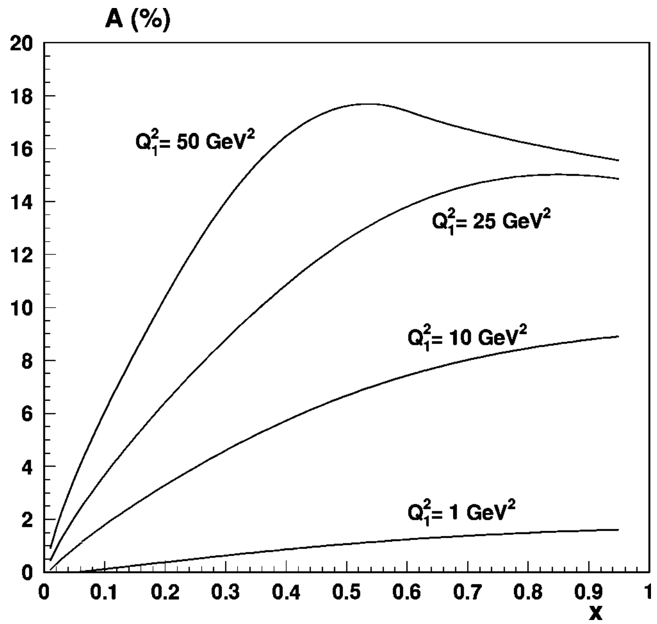


FIG. 2. The factor $\mathbf{A}(x)$ versus $x$ for $Q_1^2=1-50\ \mathrm{GeV}^2$ and $|\boldsymbol{q}/\boldsymbol{p}_2|=0.1$.

matics higher order corrections acquire odderon features, unlike in DIS, and in our approach these are parametrized by a gluon density factor.

Above we applied this approach to give a rather rough estimate for the azimuthal asymmetry of a real photon emission induced by longitudinally polarized electron in its fragmentation region. It turns out that the asymmetry is enhanced by the gluon density $zg(z,Q)$ which for $z\to 0$ appears to be $\sim 5-7\ [Q(\mathrm{GeV})]^2$. Evidently the asymmetry results from the interference between the Born-level Bethe-Heitler amplitude and that of two-loop level containing a photon-gluon fusion block. The first amplitude is real and the last one is completely imaginary. Aiming at obtaining a definite analytical result for the asymmetry we study the problem within the requirements $\boldsymbol{p}_2^2 \gg Q^2 \gg m^2$. Using this approximation we extract the gluon density factor $zg(z,Q)$. The nonenhanced terms are estimated to give a contribution of order of unity, thereby claiming the accuracy of the calculation to be of the order of 15%–20%. Just to illustrate what the asymmetry looks like we give a plot in Fig. 2 of the multiplicative factor $\mathbf{A}(x)$ in front of $\xi \sin\phi$ in Eq. (13) for the momentum transfer $Q_1^2=1-50\ \mathrm{GeV}^2$. The ratio of $|\boldsymbol{q}/\boldsymbol{p}_2|$ is kept at 0.1 for all curves in the plot. The fall of the asymmetry for high values of $x$ and $Q_1^2$ is caused by the relatively large values of $|\boldsymbol{q}|$ and, as a result, by the little bit less $\alpha_s$ values. A similar approach was developed in the papers in Ref. [4], where the authors give the formulas for the cross sections in terms of observable characteristics of diffractive scattering such as $\eta=\mathrm{Re}\,A/\mathrm{Im}\,A$. Numerical estimates show an agreement between our results.

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## APPENDIX A: EXPLICIT FORM OF $I_{\mu\nu}$

To find the contribution of the Feynman diagram containing the impact factor of a heavy photon we need to calculate the trace,

$$I_{\mu\nu}=\frac{1}{s^2}\mathrm{Tr}[(\hat{q}_-+m)B_\mu(\hat{q}_+-m)R_\nu],$$

$$B_\mu=\frac{1}{d_+}\gamma_\mu(\hat{k}-\hat{q}_++m)\hat{p}_2+\frac{1}{d_-}\hat{p}_2(\hat{q}_--\hat{k}+m)\gamma_\mu,$$

$$R_\nu=\frac{1}{d'_-}\gamma_\nu(\hat{q}_--\hat{k}'+m)\hat{p}_2+\frac{1}{d'_+}\hat{p}_2(\hat{k}'-\hat{q}_++m)\gamma_\nu, \tag{A1}$$

where $m$ is a quark mass and

$$d_\pm=k^2-2kq_\pm,\qquad d'_\pm=k'^2-2k'q_\pm.$$

It is easy to show that the gauge conditions for the on-mass-shell quarks are satisfied:

$$\overline{u(q_-)}B_\nu v(q_+)q_1^\nu=0,\quad \overline{v(q_+)}R_\mu u(q_-)k_1^\mu=0. \tag{A2}$$

Taking into account an enhancement due to the large gluon density factor $zg(z,Q)\gg 1$ one can restrict further consideration to the kinematics $\boldsymbol{p}_2^2\gg \boldsymbol{k}^2\sim \boldsymbol{q}^2$ which is thus preferable. Then it could be verified that

$$d_\pm=d'_\pm=-x_\pm\tilde{s},\quad \tilde{s}=\frac{1}{x_1}[s_1+\boldsymbol{q}_1^2]=\frac{\sigma}{x_1y_+y_-},$$

$$s_1=(q_++q_-)^2=\frac{1}{y_+y_-}[m^2+\boldsymbol{q}_t^2],$$

$$\sigma=m_*^2+\boldsymbol{q}_t^2,\qquad m_*^2=m^2+y_+y_-\boldsymbol{p}_2^2,$$

$$\boldsymbol{q}_t=\boldsymbol{q}_++y_+\boldsymbol{p}_2=-\boldsymbol{q}_--y_-\boldsymbol{p}_2,\quad y_\pm=\frac{x_\pm}{x_1}. \tag{A3}$$

Here $x_\pm$ are the energy fractions of the pair, $\boldsymbol{q}_\pm$ their components of momentum transverse to the beam axis. They obey the conservation laws

$$y_++y_-=1\quad\text{and}\quad \boldsymbol{q}_++\boldsymbol{q}_-+\boldsymbol{p}_2=0.$$

With the substitution $p_{2\mu}\to -sk_\mu^\perp/\tilde{s}$ in the quantity $B$ and, respectively, $p_{2\mu}\to -sk'^\perp_\mu/\tilde{s}$ in $R$ the tensor $I_{\mu\nu}$ can be transformed to take the following form:

$$I_{\mu\nu}=\frac{1}{4\tilde{s}^4}\mathrm{Tr}[(\hat{q}_-+m)B_{1\mu}(\hat{q}_+-m)R_{1\nu}],$$

$$B_{1\mu}=\frac{\tilde{s}}{s}\left(\frac{1}{x_+}\gamma_\mu\hat{p}\hat{k}-\frac{1}{x_-}\hat{k}\hat{p}\gamma_\mu\right)+\gamma_\mu Z,$$

$$R_{1\nu}=\frac{\tilde{s}}{s}\left(-\frac{1}{x_-}\gamma_\nu\hat{p}\hat{k}'+\frac{1}{x_+}\hat{k}'\hat{p}\gamma_\nu\right)+\gamma_\nu Z',$$

$$Z=\frac{2}{x_+x_-}\boldsymbol{rk},\quad Z'=\frac{2}{x_+x_-}\boldsymbol{rk}',\quad \boldsymbol{r}=x_1\boldsymbol{q}_t. \tag{A4}$$

Here the vectors $\boldsymbol{k}$, $\boldsymbol{k}'=\boldsymbol{k}-\boldsymbol{q}$ are pure two-dimensional ones transverse to the beam axis.

Once again one can check that the gauge conditions,

$$\overline{u}(q_-)B_{1\nu}v(q_+)q_1^\nu=0,$$

$$\overline{v}(q_+)R_{1\mu}u(q_-)k_1^\mu=0 \tag{A5}$$

are satisfied up to terms of order $\boldsymbol{k}^2/\boldsymbol{p}_2^2$.

## APPENDIX B: INTEGRATION OVER QUARK PAIR MOMENTA

Using the Sudakov parametrization of four-vectors,

$$p_1\approx\tilde{p}_1,\quad p_2=x\tilde{p}_1+\frac{\boldsymbol{p}_2^2}{xs}\tilde{p}+p_{2\perp},$$

$$q_\pm=x_\pm\tilde{p}_1+\frac{\boldsymbol{q}_\pm^2+m^2}{sx_\pm}\tilde{p}+q_{\pm\perp},$$

$$k_1=x_1\tilde{p}_1+\frac{\boldsymbol{k}_1^2}{sx_1}\tilde{p}+k_{1\perp},\quad \tilde{p}_1^2=\tilde{p}^2=0,$$

$$\tilde{p}=p-\frac{M^2}{s}p_1,\quad 2\tilde{p}\tilde{p}_1=s,\quad a_\perp\tilde{p}_1=a_\perp\tilde{p}=0, \tag{B1}$$

and the conservation law $x_++x_-=x_1$, the scalar products used can be written as follows:

$$2p_1q_\pm=\frac{1}{x_\pm}[\boldsymbol{q}_\pm^2+m^2]\equiv a_\pm,$$

$$2p_2q_\pm=\frac{1}{xx_\pm}[m^2x^2+(x\boldsymbol{q}_\pm-\boldsymbol{p}_2x_\pm)^2]\equiv a'_\pm. \tag{B2}$$

Upon averaging over the azimuthal angle of gluon momenta and using the permutation symmetry

$$x_-\leftrightarrow x_+,\qquad \boldsymbol{q}_-\leftrightarrow\boldsymbol{q}_+,$$

the quantity $J$ could be symbolically written in the following manner:

$$\frac{s\tilde{s}^4}{\boldsymbol{p}_2^2}\frac{J}{\boldsymbol{k}^2}=\frac{1}{2}(1+\mathcal{P}_\pm)[xB+C],$$

$$C=E\left(\frac{2s_t^2}{x_1^2}-\frac{4m^2\boldsymbol{r}^2}{z^2}\right)+2(p,p_2,q_-,q)\left(-\frac{2s_t}{x_1x_+z}\boldsymbol{q}_+\boldsymbol{r}+\frac{s_t^2}{x_1z}+\frac{2a_+}{z^2}\boldsymbol{r}^2-\frac{s_t^2}{x_-x_1^2}\right)-2(p,p_2,q,r)\frac{s_ta_+}{x_1z},$$

$$B=2(p,p_1,q_-,q)\left(-\frac{s_t}{x_1z}\boldsymbol{rp}_2+\frac{xs_t}{x_+x_1z}\boldsymbol{rq}_+-\frac{xs_t^2}{2x_1z}-\frac{\boldsymbol{r}^2a'_+}{z^2}+\frac{xs_t^2}{2x_-x_1^2}\right)+(p,p_1,q,r)\frac{s_ta'_+}{x_1z}+(p,p_1,p_2,r)\frac{2s_t}{x_1z}\boldsymbol{q}_-\boldsymbol{q}$$
$$+2(p,p_1,p_2,q_-)\left(\frac{s_t}{x_1z}\boldsymbol{rq}-\frac{2}{z^2}\boldsymbol{r}^2\boldsymbol{q}_+\boldsymbol{q}\right)+E\left[-\frac{2s_t(x_1+x_-)}{x_1x_-z}\boldsymbol{rq}_--\frac{s_t^2}{2z}-\frac{2}{z^2}\boldsymbol{r}^2(s_t-\boldsymbol{p}_2^2)\right]+2(p,p_2,q_-,q)$$
$$\times\left(\frac{s_t}{x_1x_+z}\boldsymbol{q}_+\boldsymbol{r}-\frac{s_t^2}{2x_1z}-\frac{a_+}{z^2}\boldsymbol{r}^2+\frac{s_t^2}{2x_-x_1^2}\right)+(p,p_2,q,r)\frac{s_ta_+}{x_1z}+s\left[2(p_1,p_2,q_-,q)\frac{x_+\boldsymbol{r}^2}{z^2}+(p_1,p_2,q,r)\frac{x_-s_t}{x_1z}\right], \tag{B3}$$

where $s_t=x_1\tilde{s}$, $z=x_+x_-$ and $\mathcal{P}_\pm$ is the permutation operator. In deriving these formulas it has been assumed that $\boldsymbol{k}^2\gg\boldsymbol{q}^2$. The structures $(\cdots)$ entering Eq. (B3) can be rewritten as follows:[1]

$$(p,p_2,q_-,q)=x(p,p_1,q_-,q)-x_-E,$$

[1]At this point one should be aware that only the transverse components of the four-vector **q** have to be taken into account.

$$s(p_1,p_2,q_-,q)=-\frac{\boldsymbol{p}_2^2}{x}(p,p_1,q_-,q)+a_-E,$$

$$s(p_1,p_2,q,r)=\frac{\boldsymbol{p}_2^2}{x}(p,p_1,r,q),$$

$$(p,p_2,q,r)=-x(p,p_1,r,q).$$

Having all the above at hand we turn to the $d^2\boldsymbol{q}_+$ integration. A set of relevant integrals reads

$$\int\frac{d^2\boldsymbol{q}_+}{\pi}\left\{\frac{1}{\sigma^2},\frac{\boldsymbol{r}^2}{\sigma^4},\frac{\boldsymbol{q}_-^i\boldsymbol{r}^j}{\sigma^3},\frac{\boldsymbol{r}^i}{\sigma^3},\frac{\boldsymbol{r}^2\boldsymbol{q}_+^i\boldsymbol{q}_-^j}{\sigma^4},\frac{(\boldsymbol{r}\boldsymbol{q}_+)\boldsymbol{q}_-^i}{\sigma^3},\frac{\boldsymbol{r}^2a'_+\boldsymbol{q}_-^i}{\sigma^4},\frac{\boldsymbol{r}^2a_+\boldsymbol{q}_-^i}{\sigma^4},\frac{a'_+\boldsymbol{r}_i}{\sigma^3},\frac{a_+\boldsymbol{r}_i}{\sigma^3},\frac{\boldsymbol{r}^2a_\pm}{\sigma^4},\frac{a_\pm}{\sigma^3},\frac{\boldsymbol{q}_+\boldsymbol{r}}{\sigma^3}\right\}$$
$$=\left\{\frac{1}{m_*^2},\frac{x_1^2}{6m_*^4},-\delta^{ij}\frac{x_1}{4m_*^2},0,-x_1^2\left(\frac{\delta_{ij}}{6m_*^2}-\boldsymbol{p}_2^i\boldsymbol{p}_2^j\frac{y_+y_-}{6m_*^4}\right),\frac{x_1\boldsymbol{p}_2^i}{2m_*^2}\left(\frac{1}{2}y_+-y_-\right),\frac{x_1^2x\boldsymbol{p}_2^i}{3m_*^2x_+}\left(\frac{y_+}{x}-y_--\frac{y_-y_+^2\boldsymbol{p}_2^2}{2x^2m_*^2}\right),\frac{x_1^2\boldsymbol{p}_2^i}{3m_*^2x_+}\right.$$
$$\left.\times\left(y_+-y_--\frac{y_-y_+^2\boldsymbol{p}_2^2}{2m_*^2}\right),-\frac{\boldsymbol{p}_2^i}{2m_*^2},-\frac{\boldsymbol{p}_2^i}{2m_*^2},\frac{x_1^2}{3x_\pm m_*^2}\left(1+\frac{y_\pm^2\boldsymbol{p}_2^2}{2m_*^2}\right),\frac{1}{2x_\pm m_*^2}\left(1+\frac{y_\pm^2\boldsymbol{p}_2^2}{m_*^2}\right),\frac{x_1}{2m_*^2}\right\}. \tag{B4}$$

Above we have discarded terms that give contributions of order $m^2/\boldsymbol{p}_2^2$ as compared with unity. The integration over $y_\pm$ becomes almost trivial in the limit $Q_1^2\gg m^2$:

$$\int_0^1\frac{dy_+}{m_*^2}\left\{1,y_\pm,y_\pm^2,y_+y_-,\frac{m^2}{m_*^2}y_-y_+\right\}=\frac{1}{\boldsymbol{p}_2^2}\{2L,L,L-1,1,0\}, \tag{B5}$$

where $L=\ln(\boldsymbol{p}_2^2/m^2)$.

## APPENDIX C: HEAVY PHOTON IMPACT FACTOR

To obtain the heavy photon IF one has to consider the $s$-channel discontinuity of the heavy photon amplitude in an external field,

$$\gamma_\mu(P_1)+A(p)\to q(q_+)+\overline{q}(q_-)+A(p'')$$
$$\to\gamma_\nu(P_2)+A(p'),$$
$$P_1^2=-Q^2,\qquad P_2^2=-Q'^2, \tag{C1}$$

which is described by the tensor

$$\Delta A_{\mu\nu}(s,t)=\frac{(4\pi\alpha)^3}{\boldsymbol{k}^2\boldsymbol{k}'^2}\left(\frac{2}{s}\right)^2N_\lambda s^4I_{\mu\nu}d\Gamma_3, \tag{C2}$$

with

$$d\Gamma_3=\frac{1}{(2\pi)^5}\frac{d^3\vec{q}_+}{2\varepsilon_+}\frac{d^3\vec{q}_-}{2\varepsilon_-}\frac{d^3\vec{p}''}{2E''}\delta^4(P_1+p-q_+-q_--p'')=\frac{d^2\boldsymbol{k}d^2\boldsymbol{q}_+dx_+}{4s(2\pi)^5x_+x_-}, \tag{C3}$$

and the quantity $N_\lambda$ given in Eq. (3). The tensor $I_{\mu\nu}$ has the form [see Eq. (A1)]

$$\frac{1}{x_+x_-}I_{\mu\nu}=(1+\mathcal{P}_{\pm})\Bigg\{\frac{x_+}{a_+a'_+}[2q_\mu q_{+\nu}+(2-4x_+)q_\nu q_{+\mu}-8x_-q_{+\mu}q_{+\nu}]+\frac{1}{a_-a'_+}(2x_+q_\mu q_{-\nu}+(-2x_++4x_+x_-)q_\nu q_{-\mu}$$
$$-2q_{-\nu}q_{+\mu}+(2-8x_+x_-)q_{+\nu}q_{-\mu})+g_{\mu\nu}\Bigg[\frac{1}{a_-a'_+}[-x_-\boldsymbol{k}^2-x_+\boldsymbol{k}'^2+x_+x_-(\boldsymbol{q}^2-Q^2-Q'^2)]$$
$$+\frac{x_+}{a_+a'_+}(x_-(Q^2+Q'^2)+x_+\boldsymbol{q}^2)\Bigg]\Bigg\}, \tag{C4}$$

with

$$a_\pm=a+\boldsymbol{q}_\pm^2, \qquad a'_\pm=b+(\boldsymbol{q}_\pm-x_\pm\boldsymbol{q})^2,$$
$$a=m^2+x_+x_-Q^2, \quad b=m^2+x_+x_-Q'^2.$$

One can argue that the gauge condition $I_{\mu\nu}=0$ for $\boldsymbol{k}=0,\boldsymbol{k}'=0$ is satisfied. Joining the denominators with the use of the Feynman trick and performing an integration over the components of the quark pair momenta transverse to the beam axis we get

$$\int\frac{d^2\boldsymbol{q}_+}{\pi a_+a'_+}=\int_0^1\frac{dy}{D_{++}}, \qquad \int\frac{d^2\boldsymbol{q}_+}{\pi a_+a'_-}=\int_0^1\frac{dy}{D_{-+}},$$
$$D_{++}=A+\boldsymbol{q}^2x_+^2y(1-y),$$
$$D_{-+}=A+y(1-y)\boldsymbol{b}^2,$$
$$\boldsymbol{b}=\boldsymbol{k}-x_+\boldsymbol{q},$$
$$A=m^2+x_+x_-[yQ'^2+(1-y)Q^2]. \tag{C5}$$

The result for the IF takes the following form (we choose only the transverse polarizations of photons $\mu\equiv i,\nu\equiv j$):

$$\tau_{ij}^\gamma=2\alpha^2\int_0^1 dx_+dx_-\delta(x_++x_--1)\int_0^1 dy\Bigg[\frac{x_+^2}{D_{++}}$$
$$\times[8x_+x_-y(1-y)q_iq_j-\boldsymbol{q}^2\delta_{ij}(1+4x_+x_-y(1-2y))]$$
$$-\frac{1}{D_{-+}}[8x_+x_-y(1-y)b_ib_j-\boldsymbol{b}^2\delta_{ij}(1+4x_+x_-y$$
$$\times(1-2y))]+4x_+x_-y(x_--x_+)q_j\left(\frac{x_+q_i}{D_{++}}+\frac{b_i}{D_{-+}}\right)$$
$$+\delta_{ij}x_+x_-(Q^2+Q'^2+4x_+x_-y(Q'^2-Q^2))$$
$$\times\left(\frac{1}{D_{-+}}-\frac{1}{D_{++}}\right)\Bigg], \tag{C6}$$

with $D_{++},D_{-+}$, and $\boldsymbol{b}$ defined in the same way as in the Eq. (C5). Once again it is clearly seen that the gauge conditions are satisfied:

$$\tau|_{\boldsymbol{k}=0}=\tau|_{\boldsymbol{k}=\boldsymbol{q}}=0.$$

It is important to note that even for the on-mass-shell photons $Q^2=Q'^2=0$ this expression differs from the one derived by Cheng and Wu [13]. The difference is found to be

$$\Delta\tau_{ij}^\gamma=\tau-\tau_{CW}$$
$$=4\alpha^2\int dx_+dy\ \ x_+x_-(1-2x_+)q_j$$
$$\times\left(\frac{x_+q_i}{D_{++}^0}+\frac{b_i}{D_{-+}^0}\right),$$
$$D_{++}^0=m^2+x_+^2y(1-y)\boldsymbol{q}^2,$$
$$D_{-+}^0=m^2+y(1-y)\boldsymbol{b}^2. \tag{C7}$$

The reason for this discrepancy is the different definition of the initial and final photon's four-momenta. Similar results were obtained in Ref. [14].


[1] A.V. Radyushkin, Phys. Lett. B **380**, 417 (1996); **385**, 333 (1996); Phys. Rev. D **56**, 5524 (1997); **59**, 014030 (1999).

[2] X. Ji, Phys. Rev. Lett. **78**, 610 (1997); Phys. Rev. D **55**, 7114 (1997); X. Ji and J. Osborne, *ibid.* **58**, 094018 (1998); P. Hoodbhoy and X. Ji, *ibid.* **58**, 054006 (1998).

[3] J.C. Collins and A. Freund, Phys. Rev. D **59**, 074009 (1999).

[4] A. Freund and M. Strikman, Phys. Rev. D **60**, 071501 (1999); L.L. Frankfurt, A. Freund, and M. Strikman, Phys. Lett. B **460**, 417 (1999); Phys. Rev. D **58**, 114001 (1998); **59**, 114001(E) (1998).

[5] M. Diehl, T. Gousset, B. Pire, and J.P. Ralston, Phys. Lett. B **411**, 193 (1997).

[6] A.V. Belitsky, D. Muller, L. Niedermeier, and A. Schaefer, Nucl. Phys. **B593**, 289 (2001).

[7] M. Vanderhaeghen, P.A. Guichon, and M. Guidal, Phys. Rev. Lett. **80**, 5064 (1998); Phys. Rev. D **60**, 094017 (1999); P.A. Guichon and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **41**, 125 (1998).

[8] I. Balitsky and E. Kuchina, Phys. Rev. D **62**, 074004 (2000).
[9] DESY workshop on Skewed Parton Distributions and Lepton-Nucleon Scattering, 2000, Hamburg, Germany.
[10] A. De Roeck, Proceedings of the workshop ''Physics with a high luminosity polarized electron ion collider,'' Bloomington, 1999, p. 225; E. Kuraev, N. Nikolaev, and B. Zakharov, JETP Lett. **68**, 696 (1998).
[11] V.N. Baier, V.S. Fadin, V.A. Khoze, and E.A. Kuraev, Phys. Rep. **78**, 293 (1981).
[12] S.J. Brodsky, L. Frankfurt, J.F. Gunion, A.H. Mueller, and M. Strikman, Phys. Rev. D **50**, 3134 (1994).
[13] H. Cheng and T. Wu, Phys. Rev. Lett. **22**, 666 (1969); L.N. Lipatov and G.V. Frolov, Yad. Fiz. **13**, 588 (1971).
[14] V.V. Davidovsky, Ukr. Phys. J. **44**, 289 (1999).